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PREFERABLE-INTERVAL APPROACHES TO THE SINGLE-SINK FIXED-CHARGE TRANSPORTATION PROBLEM

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ABSTRACT

The single-sink fixed-charge transportation problem is an important problem with applications in many areas. In this paper, a new concept of preferable interval is introduced. Properties of optimal solutions to the single-sink fixed-charge transportation problem are investigated. Solution approaches using preferable intervals are presented.

Key words: fixed-charge transportation problem, optimization, preferable interval

I. INTRODUCTION

The single-sink fixed-charge transportation (SSFCT) problem is arisen frequently in application areas of scheduling and cost control, such as facility planning, capital budgeting, resource allocation, buffer allocation, pollution control, etc. A number of applications of the SSFCT problem are discussed in Haberl et al. (1991) and Herer et al. (1996).

The SSFCT problem is to determine the amount of shipments to be made from a given set of suppliers to a single sink, such that the total demand is satisfied in a minimum cost fashion. A fixed charge and cost proportional to the quantity shipped occur when a supplier is employed. The SSFCT problem can be mathematically formulated in terms of an integer programming problem as shown below. See Herer et al. (1996).

$$\begin{aligned} \min \quad & TC = \sum_{i=1}^{M} \left(f_i d_i + F_i y_i \right) \\ \text{s.t.} \quad & \sum_{i=1}^{M} d_i = D, \\ & 0 \le d_i \le b_i y_i \qquad \text{for } 1 \le i \le M, \\ & y_i \in \{0,1\} \qquad \text{for } 1 \le i \le M, \end{aligned} \tag{P}$$

where, for $1 \le i \le M$,

b_i: the capacity of supplier i ($b_i \in \mathbb{N}$)
D: the demand magnitude ($D \in \mathbb{N}$)
M: the number of suppliers ($M \in \mathbb{N}$)
F_i: the fixed management cost incurred when using supplier i ($F_i \ge 0$)

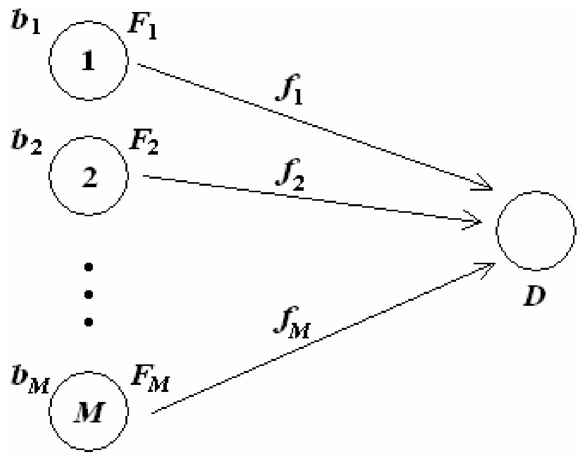

Figure 1. Graphical Depiction of the SSFCT Problem

f_i : the effective unit variable cost of supplier i ($f_i \geq 0$)
y_i : an integer variable taking the value of 1 when supplier i is used, and 0 otherwise

The solution to the problem is a vector $\boldsymbol{d} = (d_1, d_2, \cdots, d_M)$ (also called the transportation profile) for which d_i is the amount allocated to supplier i. Here d_i are nonnegative integers, $1 \leq i \leq M$. The optimal solution to the SSFCT problem is clearly not unique. The problem can be represented graphically as in Figure 1, as given in Herer et al. (1996).

The fixed-charge problem was considered as far back as Hirsch and Dantzig (1954). Balinski (1961) showed the fixed-charge transportation problem to be a special case of the fixed-charge problem and presented a heuristic solution. The SSFCT problem contains only one sink which is characterized by the demand. It has been the subject of several papers in recent years. Haberl (1991) proposed an implicit complete enumeration algorithm of complexity $O(2^M)$. Herer et al. (1996) improved the enumeration scheme of Haberl by employing domination rules and further improving lower bounds. Their procedure has complexity order $O(M2^M)$. Recently Alidaee and Kochenberger (2005) presented a dynamic programming method, which can solve the problem in $O(MD)$ time.

In this paper, a novel approach to the SSFCT problem is posed to find the optimal solutions. The concept of preferable intervals is introduced and studied. Necessary conditions of selecting suppliers are obtained in terms of preferable intervals. Several heuristic algorithms of polynomial time complexity are provided. Our further research is based on the heuristics. Enumerating algorithms using preferable intervals will be developed by adding lower bounds.

The report is organized as follows. We start with definition and properties of preferable intervals in the next section. Section 3 is devoted to properties of the optimal solutions to the SSFCT problem. In Section 4, heuristic algorithms are presented. Numeric example and summaries of finding are provided in Sections 5 and 6, respectively.

II. PREFERABLE INTERVALS OF SUPPLIERS

Standard interval notations are used in this paper.

$$[a,b]=\{x \mid x\ge a \quad and \quad x\le b\} \quad \text{and} \quad (a,b]=\{x \mid x> a \quad and \quad x\le b\}$$

In particular, $[a,a]=\{a\}$ and $(a,a]=\{\ \}$. If $a>b$, then $[a,b]=\{\ \}$ and $(a,b]=\{\ \}$. Here $\{\ \}$ stands for the empty set.

DEFINITION 2.1 *If two suppliers have the same capacities, fixed management costs, and unit variable costs, then they are called identical suppliers. If two suppliers are not identical, then they are called distinct suppliers.*

There are three different ways to assign a proper amount of work load to two suppliers, for instance, suppliers i and j. People may choose only supplier i, only supplier j, or both suppliers. We want to determine when the employment of purely one supplier, say supplier i, will yield the least cost among the three cases. Motivated by this purpose, we define the preferable interval of supplier i against supplier j as the following.

DEFINITION 2.2 *The preferable interval of supplier i against supplier j is defined as*

$$I_j^i=\begin{cases} A_j^i\cup B_j^i & \text{if suppliers } i \text{ and } j \text{ are distinct}\\ [0,b_i] & \text{if suppliers } i \text{ and } j \text{ are identical}\end{cases}$$

where

$$A_j^i=\{x\in[0,b_i] \mid F_i+f_ix<F_j+f_jx\},$$
$$B_j^i=\left(b_j, b_i\cdot \operatorname{sgn}\left(F_j+(f_j-f_i)\,b_j\right)\right],$$

and sgn(x) *is the signum function of x.*

It is clear that $I_j^i\subseteq[0,b_i]$ and $I_i^i=[0,b_i]$ for any $1\le i,j\le M$. Next, study the property of the preferable interval I_j^i of distinct suppliers i and j.

Assume that $x\in A_j^i\ne\{\ \}$. By the definition of A_j^i, it is clear that

$$F_i+f_ix<F_j+f_jx \tag{1}$$

Moreover, it can be proved that

$$F_i+f_ix<F_j+f_j\xi+F_i+f_i(x-\xi) \tag{2}$$

or, equivalently,

$$0<F_j+(f_j-f_i)\xi \tag{3}$$

for $0<\xi<x$. In order to show inequality (3), consider the following two cases. If $f_i<f_j$, since $F_i\ge 0$, then inequality (3) is trivially true. If $f_j\le f_i$, then inequality (1) and $\xi<x$ imply

$$0<F_i<F_j+(f_j-f_i)x\le F_j+(f_j-f_i)\xi$$

which is exactly (3). Hence inequality (3) is always true when $0 < \xi < x$, so is (2). Inequality (1) indicates that supplier i yields less cost than supplier j. Inequality (2) implies that using supplier i leads less cost than using both suppliers. One may thus conclude that, when $x \in A_j^i$, an employment of solely supplier i will yield the least cost among all possible assignments indicated above.

Secondly, suppose that $x \in B_j^i \neq \{ \}$. This implies that $b_j < b_i$ and

$$0 < F_j + \left(f_j - f_i\right) b_j \tag{4}$$

Since $b_j < x$, the shipment cannot be made by supplier j itself. So we need only to compare the cost by purely supplier i and the cost by both suppliers i and j; i.e. we need to ensure inequality (2) for $0 < \xi \le b_j$. By similar arguments as the proof of inequality (3) above, one can conclude that condition (4) will make (2) valid. Therefore, when $x \in B_j^i$, using only supplier i will again cost the least among the other possible assignments to both suppliers i and j.

On the other hand, when $x \le b_i$, in order to guarantee that the employment of solely supplier i yields the least cost, we need to check the following two cases. Case 1: If $x \le b_j$ then inequality (1) must hold; and (2) must hold for $0 < \xi < x$. Notice that (1) implies (2) in this situation, which leads to the fact that $x \in A_j^i$ and hence $x \in I_j^i$. Case 2: If $x > b_j$ then only inequality (2) need to be valid for $0 < \xi \le b_j$. If $f_i < f_j$, then inequality (2) is clearly true for any $0 < \xi \le b_j$; in particular, it holds when $\xi = b_j$, which is actually inequality (4). If $f_j \le f_i$, then inequality (2) holds for any $0 < \xi \le b_j$ if and only if inequality (4) is satisfied. Both situations give $x \in B_j^i$ and thus $x \in I_j^i$. Therefore, one may conclude from the two cases that x must belong to I_j^i to ensure the least cost by using only supplier i among all assignments to suppliers i and j.

We have just proved the following proposition.

PROPOSITION 2.3 *Consider distinct suppliers i and j. If the demand magnitude $x \le b_i$, then supplier i is more preferable than supplier j or any combination of both suppliers i and j in sense of cost if and only if $x \in I_j^i$.*

For simplicity of reference and comparison, we set K to be an *M*-by-*M* double array, whose *ij* th entry (i.e. the entry in the *i* th row and *j* th column) is the favorable interval of supplier i against supplier j; i.e.

$$K = \left(K_{ij}\right)_{M \times M}, \text{ where } K_{ij} = I_j^i,\ 1 \le i, j \le M$$

The double array K shall be referred to as the preferable interval matrix. Note that, to establish the preferable interval matrix, time complexity of $O(M^2)$ is required.

III. PROPERTIES OF THE OPTIMAL SOLUTIONS

The SSFCT problem is an NP-hard problem. In this section, we present properties related to the optimal solutions. The properties will be used in the development of the algorithms in Section 4.

The following two properties are analogies of properties in Herer et al. (1996).

PROPOSITION 3.1 *There exists an optimal solution to the SSFCT problem (P) where there is at most one supplier such that* $0 < d_i < b_i$*; i.e. each of the other suppliers is either not employed, or is assigned its capacity of units.*

Proof. Assume that $0 < d_{i_k} < b_{i_k}$ for suppliers i_k with $k = 1,2,\ldots,n$ and $n \geq 2$. Without loss of generality, one may assume that $f_{i_1} \leq f_{i_2} \leq \cdots \leq f_{i_n}$. Units can be passed from supplier with higher unit cost (bigger sub-index) to supplier with lower unit cost (smaller sub-index) without increasing the objective function. The property thus follows. ■

PROPOSITION 3.2 *Suppose* ***d*** *is an optimal solution to the SSFCT problem (P) with* $0 < d_i < b_i$ *for some* i*. Then* $d_k = 0$ *for all* k *with* $f_i < f_k$*.*

Proof. Show the statement by contradiction. Assume that there is a supplier k with $d_k \neq 0$ but $f_i < f_k$. One can pass units from supplier k to supplier i, which will decrease the total cost. This contradicts the fact that ***d*** is an optimal solution. ■

REMARK Note that Proposition 3.2 cannot predict the amount assigned to supplier whose unit variable cost is less than or equal to f_i. □

Suppose d_i units are awarded to supplier i, $0 < d_i \leq b_i$. Then the average unit cost of supplier i associated with d_i units is $f_i + F_i/d_i$, which is non-increasing as d_i is increasing. We thus have the following result.

PROPERTY 3.3 *The average unit cost for a supplier is non-increasing as the amount of units awarded to the supplier is increasing.*

Clearly, the smallest average unit cost of supplier i is achieved when $d_i = b_i$. Denote the cost in this situation by $e_i = f_i + F_i/b_i$, which is the *least average unit cost* of supplier i. From now on, unless otherwise stated, we shall restrict our arguments to the following assumption.

ASSUMPTION 3.4 *Assume that the suppliers are ordered according to* e_i *such that* $e_1 \leq e_2 \leq \cdots \leq e_M$.

Notice that the procedure of sorting *M* numbers requires time complexity of *O*(*M* log*M*). It follows that, in general, when Assumption 3.4 is applied, an *O*(*M* log*M*) time is involved in the algorithm.

DEFINITION 3.5 *Let b_i be the capacity of supplier i, $1 \le i \le M$, and let $b_0 = 0$. For $0 \le i \le M$, define $B_i = \sum_{k=0}^{i} b_k$, which is the capacity of the first i suppliers.*

DEFINITION 3.6 *Define n_0 to be the nonnegative integer such that $B_{n_0} \le D < B_{n_0+1}$ when $D < B_M$, and define $n_0 = M$ when $D = B_M$. Moreover, define $R = D - B_{n_0}$.*

PROPOSITION 3.7 *If $D = B_{n_0}$, then an optimal solution to the SSFCT problem (P) is $\boldsymbol{d}_0 = (b_1, b_2, \cdots, b_{n_0}, 0, \cdots, 0)$.*

Proof. If $n_0 = M$, the statement is trivial. Consider the situation of $n_0 < M$. First notice that $\boldsymbol{d}_0$ is indeed a feasible solution to problem (P). Note also that any feasible solution to problem (P) can be obtained from $\boldsymbol{d}_0$ by a sequence of transferring units from its first n_0 suppliers to the rest suppliers.

We thus can focus on a general transferring procedure from one supplier i with $i \le n_0$ to another supplier k with $k > n_0$. Initially, the average unit cost of units of supplier i is e_i. After the procedure, the average unit cost for the units remained in supplier i is not decreased, according to Property 3.3. On the other hand, the average unit cost for the units transferred into supplier k is not reduced either, since the least average unit cost of supplier k is e_k and $e_k \ge e_i$ according to Assumption 3.4.

Since each transferring procedure will not reduce the total cost of $\boldsymbol{d}_0$, one may conclude that no feasible solution to problem (P) can provide a smaller total cost than $\boldsymbol{d}_0$, which is hence optimal. ■

DEFINITION 3.8 *For $1 \le i \le M$, define $a_i = \sum_{k=1}^{i} e_k b_k \big/ B_i$, which is the least average unit cost of the first i suppliers.*

DEFINITION 3.9 *For any natural number $x \le B_M$, denote by $\Gamma(x)$ the minimized total cost of the SSFCT problem (P) with demand magnitude x. $\Gamma : N \to R$ defines a positive valued increasing function.*

REMARK It follows from Proposition 3.7 and Definitions 3.8, 3.9 that $\Gamma(B_{n_0}) = a_{n_0} B_{n_0}$. □

The following two properties are trivial from definitions.

PROPERTY 3.10 $a_1 \le a_2 \le \cdots \le a_M$.

Proof. The property is trivial from Assumption 3.4. ■

PROPERTY 3.11 *Let $\varepsilon = f_{n_0+1} + F_{n_0+1} \big/ R$. Then $e_{n_0+1} \le \dfrac{\Gamma(D) - a_{n_0} B_{n_0}}{R} \le \varepsilon$.*

Proof. Note that the least possible unit costs can be achieved for the D units are $e_1, \ldots, e_{n_0}, e_{n_0+1}$. We thus have

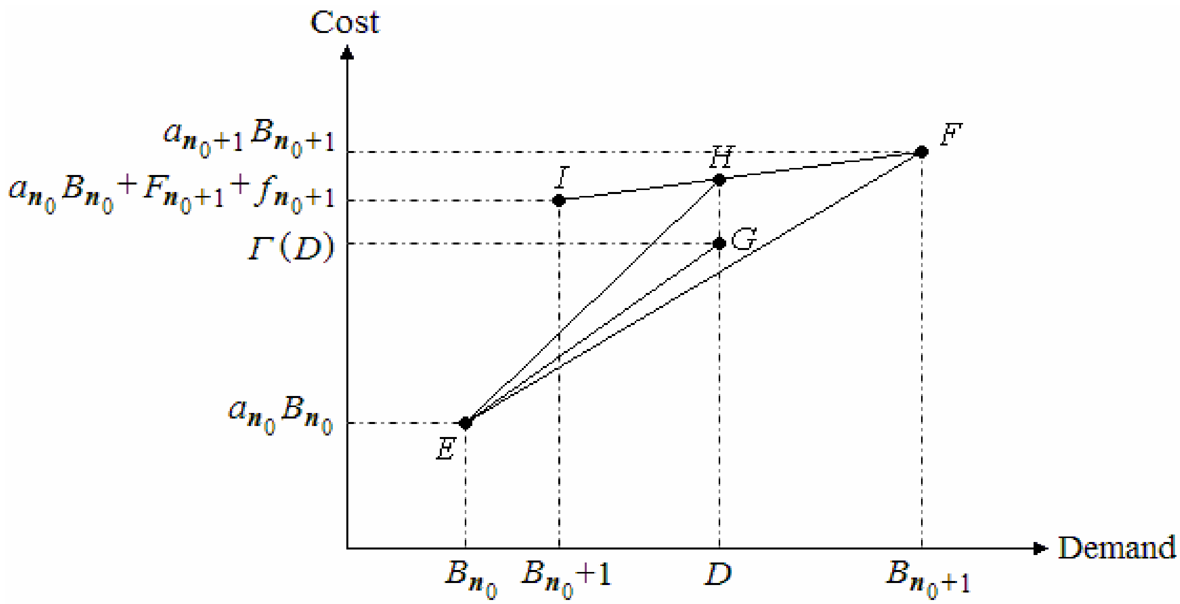

Figure 2. Geometric Representation of Proposition 3.11

$$\sum_{k=1}^{n_0} e_k b_k + e_{n_0+1}R = a_{n_0}B_{n_0} + e_{n_0+1}R \le \Gamma(D)$$

which implies the first inequality. On the other hand, the cost of the feasible solution $\boldsymbol{d}_0$ = $(b_1, b_2, \cdots, b_{n_0}, R, 0 \cdots, 0)$ is

$$\sum_{k=1}^{n_0} e_k b_k + F_{n_0+1} + f_{n_0+1}R = a_{n_0}B_{n_0} + \varepsilon R \ge \Gamma(D)$$

The second inequality thus follows. ■

REMARK Proposition 3.11 has geometric meanings. See Figure 2. Note that the vertical coordinates of points E, F, and G represent the minimized total costs corresponding to the respective demands. Therefore, the three points are on the graph of function Γ. The vertical coordinate of point I, or the cost corresponding to demand $B_{n_0}+1$, is simply obtained from the first n_0+1 suppliers by assigning full loads to the first n_0 suppliers and 1 unit to the (n_0+1)st supplier. The vertical coordinate of point H is determined from the first n_0+1 suppliers by assigning full loads to the first n_0 suppliers and R unit to the (n_0+1)st supplier. From definitions, it is easy to check that the slope of line EF is e_{n_0+1}, the slope of line EG is $\left(\Gamma(D) - a_{n_0}B_{n_0}\right)/R$, and the slope of line EH is ε. Proposition 3.11 is hence clearly illustrated by Figure 2. It is worth to point out that the slope of line FI is f_{n_0+1}. Moreover, line EH intersects line FI at point H. □

PROPOSITION 3.12 *Suppose $\boldsymbol{d}$ is an optimal solution to the SSFCT problem (P). Let $\varepsilon = f_{n_0+1} + F_{n_0+1}/R$. Then $e_i > \varepsilon$ implies that $d_i = 0$.*

Proof. Prove the property by contradiction. Assume that an optimal solution ***d*** has $d_{i_k} > 0$ and $e_{i_k} > \varepsilon$ for $k = 1,2,\ldots,n$. Compare solutions ***d*** and ***d***$_0$ = $(b_1, b_2, \cdots, b_{n_0}, R, 0 \cdots, 0)$. Since $e_{n_0+1} \le \varepsilon$, one concludes from Assumption 3.4 that $i_k > n_0 + 1$, $k = 1,2,\ldots,n$. That is the group of suppliers i_k, $k = 1,2,\ldots,n$ and the group of the first $n_0 + 1$ suppliers are disjoint. Thus, ***d***$_0$ can be obtained from ***d*** by transferring units from the group of suppliers i_k, $k = 1,2,\ldots,n$, to the first $n_0 + 1$ suppliers. Notice that e_{i_k} is the least possible unit cost of supplier i_k, it follows that the average cost of the units transported by each supplier i_k is greater than ε. But $e_i \le e_{n_0+1} \le \varepsilon$ for $1 \le i \le n_0 + 1$. Therefore, when units from a supplier i_k are passed to one of the first $n_0 + 1$ suppliers, the unit cost is reduced. Hence, the total cost decreases as a result. This contradicts the assumption that ***d*** is an optimal solution. The desired result thus follows. ■

REMARK Propositions 3.2 and 3.12 provide sufficient conditions of an optimal solution to reject suppliers. □

PROPOSITION 3.13 *Suppose **d** is an optimal solution to the SSFCT problem (P). Then* $d_i \in I_j^i$ *for all* i *with* $d_i \ne 0$ *and* j *with* $d_j = 0$.

Proof. If suppliers i and j are distinct, then this property is straightforward from Proposition 2.3 and the fact that ***d*** is an optimal solution to problem (P). If suppliers i and j are identical, then Definition 2.2 implies $I_j^i = [0, b_i]$, and hence $d_i \in I_j^i$. ■

PROPOSITION 3.14 *Suppose **d** is an optimal solution to the SSFCT problem (P) which satisfies Proposition 3.1 with* $0 < d_i < b_i$ *for some* i. *Then* $d_i + b_j \notin I_j^i$ *for all* j *with* $d_j = b_j$.

Proof. If $d_i + b_j \in I_j^i$ for some j with $d_j = b_j$, then $d_i + b_j \le b_i$. The total cost can be reduced by reassigning all units of supplier j to supplier i, according to Proposition 2.3. But this contradicts the assumption of optimal solution. ■

REMARK Proposition 3.13 indicates a necessary condition of an optimal solution to reject a supplier. On the other hand, Proposition 3.14 gives a necessary condition of an optimal solution for employing a supplier. □

IV. APPROACHES FOR SELECTING SUPPLIERS

In this section, we present several approaches for selecting suppliers. The schemes are developed based on properties studied in Section 3. All algorithms below are under Assumption 3.4 and the condition $B_M \ge D$.

ALGORITHM 4.1 (Heuristic)

Step 1 Set ***d*** = **0**, $x = D$, $j = 1$;

Step 2 Set $d_j = \min\{b_j, x\}$, $x = x - b_j$;

Step 3 If $x > 0$, then set $j = j+1$ and go to Step 2;
Else stop.

Algorithm 4.1 is a simple heuristic of time complexity *O*(*M* log*M*), which provides the feasible solution $\boldsymbol{d}_0 = (b_1, b_2, \cdots, b_{n_0}, R, 0 \cdots, 0)$ with n_0 and R as defined in Section 3. In the algorithm, x represents unallocated demand. Note that at most one entry of $\boldsymbol{d}_0$ satisfies $0 < d_i < b_i$; i.e. when $i = n_0 + 1$. Moreover, from Proposition 3.7, it follows that, if $D = B_{n_0}$, then an optimal solution can be obtained from Algorithm 4.1.

Algorithm 4.1 can be improved according to Proposition 3.14. A resulting algorithm is the following.

ALGORITHM 4.2 (Heuristic)
Step 1 Set $\boldsymbol{d} = \boldsymbol{d}_0$, where $\boldsymbol{d}_0$ is the solution from Algorithm 4.1;
Step 2 If $n_0 > 0$, then set $i_0 = n_0 + 1$, $j = n_0$;
Else stop;
Step 3 If $d_j = b_j$ and $d_{i_0} + b_j \in I_j^{i_0}$, then set $d_j = 0$, $d_{i_0} = d_{i_0} + b_j$;
Step 4 Set $j = j - 1$. If $j > 0$, then go to Step 3; else set $x = d_{i_0}$;
Step 5 Choose a supplier k from those suppliers with $d_k = b_k$ such that f_k is maximum;
Step 6 If $f_k > f_{i_0}$, then set $d_{i_0} = \min\{b_{i_0}, x + b_k\}$, $d_k = x + b_k - d_{i_0}$;
Else stop;
Step 7 If $d_k = 0$, then set $x = x + b_k$, go to Step 5;
Step 8 If $d_k \neq b_k$, then set $i_0 = k$, $j = n_0 + 1$, go to Step 3;
Else stop.

Algorithm 4.2 restricts the process in the first $n_0 + 1$ suppliers. It first checks the condition indicated in Proposition 3.14, then reduces the corresponding total cost by passing units from suppliers with higher unit costs to those with lower unit costs. Each mid step of the solution vector $\boldsymbol{d}$ has at most one entry with $0 < d_i < b_i$. Note that if the unit variable costs of the first n_0 suppliers were sorted, then it takes no work time to find the maximum f_k in each visit of Step 5. It takes $O(n_0 \log n_0)$, or at most *O*(*M* log*M*) time to sort n_0 numbers. Note also that there are two layers of loops in the algorithm which require $O(n_0^2)$, or at most $O(M^2)$ processing time. Therefore, Algorithm 4.2 is in general a heuristic scheme of complexity $O(M^2)$.

By Propositions 3.13 and 3.14, a global heuristic algorithm can be obtained.

ALGORITHM 4.3 (Heuristic)
Step 1 Set $\boldsymbol{d} = \boldsymbol{d}_0$, where $\boldsymbol{d}_0$ is the solution vector from Algorithm 4.2;
Step 2 Set i_0 the supplier with $0 < d_{i_0} < b_{i_0}$;
If there is not such a supplier, set $i_0 = n_0 + 1$;

Step 3 Set i the first supplier with $d_i = b_i$, j the first supplier with $d_j = 0$, and $m = 0$;

Step 4 If $d_j \in I_{i_0}^j$, then set $k = i_0$, go to Step 9;

Step 5 If $d_j \in I_i^j$, then set $k = i$, go to Step 9;

Step 6 If there is a next supplier i with $d_i = b_i$, then set i the new supplier, go to Step 5;

Step 7 If there is a next supplier j with $d_j = 0$, then set j the new supplier, go to Step 4;

Step 8 If $m = 1$, then go to Step 3;
Else stop;

Step 9 Set $d_j = d_k$, $d_k = 0$, $m = 1$;

Step 10 Starting from the first supplier s with $d_s = b_s$ and in order: if $d_j + d_s \in I_s^j$, then set $d_j = d_j + d_s$ and $d_s = 0$;

Step 11 Do this step till supplier j is fully loaded: choose a supplier t from those suppliers with $d_t = b_t$ such that f_t is maximum, then pass units from supplier t to supplier j;
Go to Step 2.

Algorithm 4.3 applies Proposition 3.13 in Steps 4 through 7, and applies Proposition 3.14 in Steps 9 through 11. Since preferable intervals are frequently used in the scheme, it is convenient to generate the double array K indicated in Section 2 in the preprocessing. Note that there are three layers of loops in the algorithm which require $O(M^3)$ processing time.

V. COMPUTATIONAL EXAMPLE

Consider an example with 10 suppliers and a sink demand $D = 17$. The supplier data are shown in Table 1.

	F_i	b_i	f_i	e_i
1	41	4	13	23.25
2	59	3	8	27.67
3	65	3	13	34.67
4	68	3	14	36.67
5	53	2	11	37.50
6	92	3	8	38.67
7	67	2	9	42.50
8	67	2	11	44.50
9	69	2	13	47.50
10	84	2	7	49.00

The initial solution of Algorithm 4.3, i.e. solution from Algorithm 4.2 is $(4,3,3,2,2,3,0,0,0,0)$ with $i_0 = 4$. Since $2 \in I_4^7 = [0,2]$, assignments to supplier 4 are transported to the next output in Algorithm 4.3 is $(4,3,3,0,2,3,2,0,0,0)$. This time the first supplier with no work load is supplier 4. Since $3 \in I_6^4 = [0,3]$, we need to pass 3 units from supplier 6 to supplier 4, according to Algorithm 4.3. The corresponding result is $(4,3,3,3,2,0,2,0,0,0)$. This turns out to be the optimal solution.

VI. Conclusions

Thanks to the support of the Texas Center Research Fellows Grant Program 2005-2006, we have started research in the single-sink fixed-charge transportation problem. In this note, a new concept of preferable intervals has been introduced and studied. Properties of optimal solutions to the problem are investigated. Heuristic algorithms using preferable intervals are presented. Our future research is targeted at branch-and-bound enumerating algorithms using preferable intervals.

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